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Corollary:

S_n is not solvable for $n \geq 5$.

Proof: A_n is a non commutative group, so $A_n' \neq \{e\}$, e is identity. Moreover A_n is simple (i.e. no proper normal subgroup) and A_n' is a normal subgroup of A_n . So $A_n' = A_n$.

Again $A_n^{(2)} = (A_n')' = A_n' = A_n$

In general $A_n^{(k)} = A_n \neq \{e\}$ for every positive integer k . Hence A_n is not solvable. Therefore S_n is not solvable as subgroup of a solvable group is solvable.

Maximal normal subgroup:

A normal subgroup H of a group G is said to be maximal normal subgroup if $H \neq G$ and there does not exist a normal subgroup K such that $H \subset K \subset G$.

For a normal subgroup H of G , subgroup of G/H is of the form K/H where K is a subgroup of G containing H . Again K/H is a normal subgroup of G/H iff K is a normal subgroup of G (containing H). K/H is a proper subgroup of G/H iff $H \subset K \subset G$.

"What seems to us as bitter trials are often blessings in disguise." - Oscar Wilde



Thus H is a maximal normal subgroup of G iff $H \neq G$ and G/H has no proper normal subgroup i.e. G/H is simple.

Exm. Let $G = \langle a \rangle$ be of order 10. Then $H = \langle a^2 \rangle$ is of order 5. So G/H is of order 2 and hence simple. (A ~~sub~~group of prime order is simple). So H is a maximal ^{normal} subgroup of G .

Exm: If G is a simple group then $\{e\}$ is a maximal normal subgroup of G .

Exm: In S_n ($n > 2$), A_n is a maximal normal subgroup.

Refinement: Let $G = G_0 \supseteq G_1 \supseteq G_2 \supseteq \dots \supseteq G_m = \{e\}$ is called a subnormal series of G . A subnormal series $G = H_0 \supseteq H_1 \supseteq \dots \supseteq H_n = \{e\}$ is called a refinement of the former series if every G_i is one of H_j 's.